Estimation ROC curves and their corresponding ideal observers

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ABSTRACT

The LROC curve may be generalized in two ways. We can replace the location of the signal with an arbitrary set of parameters that we wish to estimate. We can also replace the binary function that determines whether an estimate is correct by a utility function that measures the usefulness of a particular estimate given the true parameter set. The expected utility for the true-positive detections may then be plotted versus the false-positive fraction as the detection threshold is varied to generate an estimation ROC curve (EROC). Suppose we run a 2AFC study where the observer must decide which image has the signal and then estimate the parameter set. Then the average value of the utility on those image pairs where the observer chooses the correct image is an estimate of the area under the EROC curve (AEROC). The ideal LROC observer may also be generalized to the ideal EROC observer, whose EROC curve lies above those of all other observers. When the utility function is non-negative the ideal EROC observer shares many properties with the ideal ROC observer, which can simplify the calculation of the ideal AEROC. When the utility function is concave the ideal EROC observer makes use of the posterior mean estimator. Other estimators that arise as special cases include maximum *a posteriori* estimators and maximum likelihood estimators. Multiple signals may be accomodated in this framework by making the number of signals one of the parameters in the set to be estimated.

Keywords: Image quality, estimation, ROC analysis

1. INTRODUCTION

The evaluation of imaging systems based on observer performance for a combined detection and estimation task has been studied extensively when the parameter to be estimated is the location of a signal. One figure of merit in this situation is ALROC, the area under the localization receiver operating characteristic (LROC) curve. Recently Khurd and Gindi[1] have determined the ideal LROC observer, whose LROC curve lies above those of all other observers for the given task. Of course, this also implies that the ideal LROC observer maximizes ALROC for the given task.

We are proposing a general framework, the Estimation ROC curve (EROC), for the evaluation of observers on combined detection and estimation tasks. We define the EROC curve for the detection of a signal and the estimation of a set of signal parameters. This curve is a straightforward generalization of the LROC curve. We show how the area under the EROC curve (AEROC) is related to a 2AFC test. Finally we formulate the EROC ideal observer, whose EROC curve lies above those of all other observers for the given task, and study its properties. This ideal-EROC observer is again an easy generalization of the ideal-LROC observer described by Khurd and Gindi.[1]

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2. THE EROC CURVE

An observer for a combined detection and estimation task is provided a data vector $\mathbf{g}$ that is drawn from either a signal absent probability distribution $pr(\mathbf{g}|H_0)$ or a signal present probability distribution $pr(\mathbf{g}|\boldsymbol{\theta}, H_1)$. The vector $\boldsymbol{\theta}$ is a parameter vector associated with the signal. The observer must decide whether the signal is present or not. If we assume that the observer is not subject to internal noise, this decision can be reduced to the comparison of a test statistic $T(\mathbf{g})$ with a threshold T_0. If $T(\mathbf{g}) > T_0$, then the observer declares the signal to be present. Otherwise the signal is declared to be absent. If the observer decides that the signal is present, then an estimate $\hat{\boldsymbol{\theta}}(\mathbf{g})$ of the parameter $\boldsymbol{\theta}$ must be produced.

The utility of the estimate $\hat{\boldsymbol{\theta}}(\mathbf{g})$ is denoted by $u\left[\hat{\boldsymbol{\theta}}(\mathbf{g}), \boldsymbol{\theta}\right]$ when the signal is actually present and the true parameter vector is $\boldsymbol{\theta}$. In general we would expect this function to have high values when the estimate is close to the true parameter vector, and low values when it is far from the true parameter vector.

For the EROC curve we define the false-positive fraction in the usual way as

$$P_{FP}(T_0) = \int p(\mathbf{g}|H_0)\, step\left[T(\mathbf{g}) - T_0\right] d\mathbf{g}. \tag{1}$$

This number is the abscissa of the point on the EROC curve corresponding to the threshold value T_0. For the ordinate we use the expected utility for those data vectors where the utility function is defined, i.e. the true-positive fraction. Using the prior distribution $pr(\boldsymbol{\theta})$ on the signal parameter, this expected utility is

$$U_{TP}(T_0) = \int\int pr(\boldsymbol{\theta})\, pr(\mathbf{g}|\boldsymbol{\theta}, H_1)\, u\left[\hat{\boldsymbol{\theta}}(\mathbf{g}), \boldsymbol{\theta}\right] step\left[T(\mathbf{g}) - T_0\right] d\boldsymbol{\theta} d\mathbf{g}. \tag{2}$$

Note that we have to average over the signal parameter vector since we do not know what it is. The plot of $U_{TP}(T_0)$ versus $P_{FP}(T_0)$ as the threshold is varied is the EROC curve. Each point on the EROC curve corresponds to the expected utility of our estimate for the true-positive cases at a given false-positive fraction.

2.1. The area under the EROC curve

The area under the EROC curve is given by

$$AEROC = \int_0^1 U(T_0)\, dP_{FP}(T_0). \tag{3}$$

This quantity can be used as a figure of merit for the observer on the combined detection and estimation task. By taking the derivative of the false-positive fraction with respect to the threshold, we arrive at an alternative expression for the AEROC:

$$AEROC = \langle U\left[T(\mathbf{g})\right]\rangle_{\mathbf{g}|H_0}. \tag{4}$$

One useful property of the AEROC as a figure of merit is that it can be computed from a 2AFC test. This fact can be derived from Equation 4 by writing out the expected utility inside the angle brackets:

$$AEROC = \left\langle \left\langle u\left[\hat{\boldsymbol{\theta}}(\mathbf{g}'), \boldsymbol{\theta}\right] step\left[T(\mathbf{g}') - T(\mathbf{g})\right] \right\rangle_{\mathbf{g}', \boldsymbol{\theta}|H_1} \right\rangle_{\mathbf{g}|H_0}. \tag{5}$$

For the 2AFC test the observer is shown many pairs of data vectors, with each pair consisting of a signal absent case and a signal present case. The observer must first decide which of the pair of data vectors is from the signal present ensemble, and then estimate the parameter for that vector. The average utility of the estimates for the pairs where the correct data vector was chosen is then an estimate of this observer's AEROC.

3. THE IDEAL EROC OBSERVER

Another useful property of the EROC curve is that there is an ideal EROC observer, one whose EROC curve lies above all others for the given probability distributions. To formulate this ideal observer we first define a conditional likelihood ratio as

$$\Lambda\left(\mathbf{g}|\boldsymbol{\theta}\right)=\frac{pr\left(\mathbf{g}|\boldsymbol{\theta},H_1\right)}{pr\left(\mathbf{g}|H_0\right)}. \tag{6}$$

The ideal EROC observer test statistic is given by the maximum of a likelihood-ratio-weighted average of the utility function:

$$T_I\left(\mathbf{g}\right)=\max_{\boldsymbol{\theta}'}\left\{\int pr\left(\boldsymbol{\theta}\right)\Lambda\left(\mathbf{g}|\boldsymbol{\theta}\right)u\left(\boldsymbol{\theta}',\boldsymbol{\theta}\right)d\boldsymbol{\theta}\right\}. \tag{7}$$

This integral could also be viewed as a utility weighted average of the likelihood ratio. The ideal EROC observer estimator is actually computed along with the test statistic:

$$\hat{\boldsymbol{\theta}}_I\left(\mathbf{g}\right)=\arg\max_{\boldsymbol{\theta}'}\left\{\int pr\left(\boldsymbol{\theta}\right)\Lambda\left(\mathbf{g}|\boldsymbol{\theta}\right)u\left(\boldsymbol{\theta}',\boldsymbol{\theta}\right)d\boldsymbol{\theta}\right\}. \tag{8}$$

The proof that these expressions give the ideal EROC observer is an easy adaptation of the proof presented by Khurd and Gindi for the ideal LROC observer. We simply replace $\mathbf{r}$ with $\boldsymbol{\theta}$, and $circ\left[\frac{|\hat{\mathbf{r}}(\mathbf{g})-\mathbf{r}|}{R}\right]$ with $u\left[\hat{\boldsymbol{\theta}}\left(\mathbf{g}\right),\boldsymbol{\theta}\right]$, in their LROC expressions, and then follow the same steps to derive the ideal EROC observer.

3.1. Alternate expressions for the ideal AEROC

One interesting result of the definition of the ideal EROC observer is that the AEROC for this observer can be computed by sampling independent pairs from the signal-absent ensemble :

$$AEROC_I=\left\langle\left\langle T_I\left(\mathbf{g}'\right)step\left[T_I\left(\mathbf{g}'\right)-T_I\left(\mathbf{g}\right)\right]\right\rangle_{\mathbf{g}'|H_0}\right\rangle_{\mathbf{g}|H_0}. \tag{9}$$

To see why this equation is true note that

$$T_I\left(\mathbf{g}'\right)=\int pr\left(\boldsymbol{\theta}\right)\Lambda\left(\mathbf{g}'|\boldsymbol{\theta}\right)u\left(\hat{\boldsymbol{\theta}}_I\left(\mathbf{g}'\right),\boldsymbol{\theta}\right)d\boldsymbol{\theta}. \tag{10}$$

This expression gives us

$$\left\langle T_I\left(\mathbf{g}'\right)step\left[T_I\left(\mathbf{g}'\right)-T_I\left(\mathbf{g}\right)\right]\right\rangle_{\mathbf{g}'|H_0}=\left\langle u\left[\hat{\boldsymbol{\theta}}\left(\mathbf{g}'\right),\boldsymbol{\theta}\right]step\left[T\left(\mathbf{g}'\right)-T\left(\mathbf{g}\right)\right]\right\rangle_{\mathbf{g}',\boldsymbol{\theta}|H_1}. \tag{11}$$

On the right in this equation is the inner expectation in the 2AFC expression for the ideal AEROC in Equation 5. Since the outer expectations in the two expressions for the ideal AEROC are the same, this shows their equivalence.

If the distribution of the ideal EROC test statistic under the signal absent hypothesis is known, then we may use

$$AEROC_I=\left\langle\left\langle T_I step\left(T_I-T_I'\right)\right\rangle_{T_I'|H_0}\right\rangle_{T_I|H_0}. \tag{12}$$

This equation is similar to an expression for the AUC of the ideal observer for a pure detection task,[2]

$$AUC_I=\left\langle\left\langle\Lambda step\left(\Lambda-\Lambda'\right)\right\rangle_{\Lambda'|H_0}\right\rangle_{\Lambda|H_0}, \tag{13}$$

where $\Lambda\left(\mathbf{g}\right)$ is the likelihood ratio. This last expression leads to other equalities and inequalities that relate the ideal AUC to various moments of the likelihood ratio under the signal-absent hypothesis. Analogous relations can be derived for the AEROC and signal-absent moments of T_I.

For example, evaluating the step function immediately gives

$$AEROC_I = \int_{-\infty}^{\infty} \int_{T_I'}^{\infty} T_I pr\left(T_I|H_0\right) pr\left(T_I'|H_0\right) dT_I dT_I' \tag{14}$$

This equation is equivalent to

$$\langle T_I \rangle_{T_I|H_0} - AEROC_I = \int_{-\infty}^{\infty} \int_{-\infty}^{T_I'} T_I pr\left(T_I|H_0\right) pr\left(T_I'|H_0\right) dT_I dT_I' \tag{15}$$

Symmetric versions of these two equations can be derived also:

$$AEROC_I = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max\left(T_I, T_I'\right) pr\left(T_I|H_0\right) pr\left(T_I'|H_0\right) dT_I dT_I' \tag{16}$$

and

$$\langle T_I \rangle_{T_I|H_0} - AEROC_I = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \min\left(T_I, T_I'\right) pr\left(T_I|H_0\right) pr\left(T_I'|H_0\right) dT_I dT_I' \tag{17}$$

Either of the first pair of equations, combined with either of the last pair, leads to the bounds

$$\frac{1}{2} \langle T_I \rangle_{T_I|H_0} \leq ALROC_I \leq \langle T_I \rangle_{T_I|H_0} \tag{18}$$

The mean value of the ideal decision statistic under the signal absent hypothesis is also bounded as long as the utility function is:

$$\langle T_I \rangle_{T_I|H_0} \leq \max_{\boldsymbol{\theta}', \boldsymbol{\theta}} u\left(\boldsymbol{\theta}', \boldsymbol{\theta}\right) \tag{19}$$

We will assume that this moment is bounded in most of what follows.

3.2. Positive utility functions

If the utility function is positive, then the ideal decision test statistic will be positive and we may define an equivalent test statistic by

$$t = \ln\left(T_I\right) \tag{20}$$

Then the ideal AEROC is given by

$$AEROC_I = \left\langle \langle \exp\left(t'\right) step\left(t' - t\right) \rangle_{t|H_0} \right\rangle_{t'|H_0} \tag{21}$$

The following chain of equalities leads to an expression for the ideal AEROC in terms of complex moments of T_I. We start with the characteristic function

$$\psi_t\left(\omega\right) = \langle \exp\left(-2\pi i \omega t\right) \rangle_{t|H_0} \tag{22}$$

and write the ideal AEROC as an integral in frequency space

$$AEROC_I = \int_{-\infty}^{\infty} \psi_t\left(\omega\right) \left[\psi_t\left(\omega - \frac{1}{2\pi i}\right)\right]^* \left[\frac{1}{2}\delta\left(\omega\right) + \mathcal{P}\frac{1}{2\pi i \omega}\right] d\omega \tag{23}$$

Evaluating the delta-function gives

$$AEROC_I = \frac{1}{2}\left[\psi_t\left(-\frac{1}{2\pi i}\right)\right]^* + \frac{1}{2\pi i}\mathcal{P}\int_{-\infty}^{\infty} \psi_t\left(\omega\right) \left[\psi_t\left(\omega - \frac{1}{2\pi i}\right)\right]^* \frac{d\omega}{\omega} \tag{24}$$

In terms of the ideal test statistic T_I this expression becomes

$$AEROC_I = \frac{1}{2} \langle T_I \rangle_{T_I|H_0} + \frac{1}{2\pi i}\mathcal{P}\int_{-\infty}^{\infty} \langle T_I^{-2\pi i \omega} \rangle_{T_I|H_0} \langle T_I^{-2\pi i \omega + 1} \rangle_{T_I|H_0}^* \frac{d\omega}{\omega} \tag{25}$$

If we let $\beta = -2\pi\omega$ the integral can be written as

$$AEROC_I = \frac{1}{2}\langle T_I\rangle_{T_I|H_0} - \frac{1}{2\pi i}\mathcal{P}\int_{-\infty}^{\infty}\left\langle T_I^{i\beta}\right\rangle_{T_I|H_0}\left\langle T_I^{1-i\beta}\right\rangle_{T_I|H_0}\frac{d\beta}{\beta}. \tag{26}$$

This last integral can be viewed as a contour integral along the imaginary axis. We want to shift the contour one-half unit to the right in order to dispense with the principal value. The integrand is analytic on the strip between these two contours due to two inequalities. The first inequality is

$$\left|\left\langle T_I^{x+iy}\right\rangle_{T_I|H_0}\right| \leq \langle T_I^x\rangle_{T_I|H_0}, \tag{27}$$

which is true for any x and y, and the second inequality is

$$\langle T_I^x\rangle_{T_I|H_0} \leq 1 + \langle T_I\rangle_{T_I|H_0}, \tag{28}$$

which is true for $0 \leq x \leq 1$. When we shift the contour, $i\beta$ is replaced by $\frac{1}{2} + i\alpha$. After taking into account the pole at the origin and then keeping only the real part we arrive at

$$AEROC_I = \langle T_I\rangle_{T|H_0} - \frac{1}{4\pi}\int_{-\infty}^{\infty}\left\langle T_I^{\frac{1}{2}+i\alpha}\right\rangle_{T_I|H_0}\left\langle T_I^{\frac{1}{2}-i\alpha}\right\rangle_{T_I|H_0}\frac{d\alpha}{\alpha^2+\frac{1}{4}} \tag{29}$$

This equation can also be written as

$$AEROC_I = \langle T_I\rangle_{T_I|H_0} - \frac{1}{4\pi}\int_{-\infty}^{\infty}\left|\left\langle T_I^{\frac{1}{2}-i\alpha}\right\rangle_{T_I|H_0}\right|^2\frac{d\alpha}{\alpha^2+\frac{1}{4}} \tag{30}$$

This integral expression for the ideal AEROC can be used to relate this quantity to Fisher information integrals when the signal is weak.[3] This relation will be detailed elsewhere.

There are several inequalities that can be derived from the various expressions for the ideal AEROC presented above. We will confine ourselves here to one example that follows easily from the last integral above:

$$AEROC_I \geq \langle T_I\rangle_{T_I|H_0} - \frac{1}{2}\left\langle T_I^{\frac{1}{2}}\right\rangle_{T_I|H_0}^2 \tag{31}$$

This lower bound is an improvement over the one given above since the Schwartz inequality can be used to show that $\left\langle T_I^{\frac{1}{2}}\right\rangle_{T_I|H_0}^2 \leq \langle T_I\rangle_{T_I|H_0}$. There is a similar lower bound for the AUC of the ideal observer in a pure detection task which is related to $G(0)$, the likelihood generating function evaluated at the origin.[2]

The quantity $G(0)$ can also be used to approximate the ideal observer AUC in the detection task. There is a corresponding approximation for the ideal AEROC for positive utility functions that can be derived from the last integral expression for $AEROC_I$. In this case we define $G(0)$ using

$$\langle T_I\rangle_{T_I|H_0}\exp\left[-\frac{1}{2}G(0)\right] = \left\langle T_I^{\frac{1}{2}}\right\rangle_{T_I|H_0}^2 \tag{32}$$

and the approximation is given by

$$AEROC_I \approx \langle T_I\rangle_{T_I|H_0}\left\{\frac{1}{2} + \frac{1}{2}erf\left[\frac{1}{2}\sqrt{2G(0)}\right]\right\}. \tag{33}$$

The basic fact that leads to this approximation is that the normalized ideal AEROC test statistic

$$\Lambda = \frac{T_I}{\langle T_I\rangle_{T_I|H_0}} \tag{34}$$

is a likelihood ratio, albeit for an artificially constructed detection task. Details of this derivation will be provided elsewhere. In summary then, all of the inequalities and approximations for ideal AUC observers[4, 5] have their counterparts for ideal AEROC observers when the utility function is non-negative.

3.3. Special Cases

The ideal EROC observer uses maximum a priori (MAP) estimation when the utility function is $\delta\left(\boldsymbol{\theta}'-\boldsymbol{\theta}\right)$. In this case we have

$$T_I(\mathbf{g}) = \max_{\boldsymbol{\theta}}\left\{\frac{pr(\mathbf{g},\boldsymbol{\theta}|H_1)}{pr(\mathbf{g}|H_0)}\right\} \tag{35}$$

$$\hat{\boldsymbol{\theta}}_I(\mathbf{g}) = \arg\max_{\boldsymbol{\theta}}\left\{pr(\boldsymbol{\theta})\,pr(\mathbf{g}|\boldsymbol{\theta},H_1)\right\}. \tag{36}$$

Notice that the decision statistic involves the signal absent distribution on the data vector. The δ utility function is not bounded but we have

$$\langle T_I\rangle_{T_I|H_0} = \int \max_{\boldsymbol{\theta}}\left\{pr(\mathbf{g},\boldsymbol{\theta}|H_1)\right\}d\mathbf{g} \tag{37}$$

which is often a finite quantity. Of course this utility function is rather demanding, but these results do indicate that MAP estimation is close to optimal in the EROC sense when close tolerances are required for the parameter estimation.

If in additon to the δ utility function we also have a flat prior on the parameters, then the ideal EROC observer uses maximum likelihood (ML) estimation

$$T_I(\mathbf{g}) = \max_{\theta}\left\{\Lambda(\mathbf{g}|\boldsymbol{\theta})\right\} \tag{38}$$

$$\hat{\boldsymbol{\theta}}_I(\mathbf{g}) = \arg\max_{\boldsymbol{\theta}}\left\{pr(\mathbf{g}|\boldsymbol{\theta},H_1)\right\} \tag{39}$$

In this case the decision statistic is the conditional likelihood ratio at the estimated parameter value. The implication here is that ML estimation and likelihood windowing are close to optimal in the EROC sense when we have close tolerances for our estimates and complete ignorance about the true parameter values.

When the utility function is symmetric and concave, the posterior mean estimator[6]

$$\hat{\boldsymbol{\theta}}_I(\mathbf{g}) = \int \boldsymbol{\theta}\,pr(\boldsymbol{\theta}|\mathbf{g})\,d\boldsymbol{\theta} \tag{40}$$

is the ideal EROC estimator with the corresponding test statistic

$$T_I(\mathbf{g}) = \int pr(\boldsymbol{\theta})\,\Lambda(\mathbf{g}|\boldsymbol{\theta})\,u\left(\hat{\boldsymbol{\theta}}_I(\mathbf{g}),\boldsymbol{\theta}\right)d\boldsymbol{\theta}. \tag{41}$$

The concavity of the utility function implies that it cannot be positive unless the range of each parameter is bounded.

Finally we consider the case of a Gaussian probablity distribution for the data

$$pr(\mathbf{g}|H_0) = \frac{1}{\sqrt{(2\pi)^M\det(\mathbf{K})}}\exp\left[-\frac{1}{2}(\mathbf{g}-\mathbf{b})^\dagger\mathbf{K}^{-1}(\mathbf{g}-\mathbf{b})\right] \tag{42}$$

with the signal in the mean:

$$pr(\mathbf{g}|\boldsymbol{\theta},H_1) = pr(\mathbf{g}-\mathbf{s}_{\boldsymbol{\theta}}|H_0). \tag{43}$$

With the δ utility function and a flat prior, the decision statistic in this case is given by

$$T_I(\mathbf{g}) = \max_{\boldsymbol{\theta}}\left\{\exp\left[\mathbf{s}_{\boldsymbol{\theta}}^\dagger\mathbf{K}^{-1}(\mathbf{g}-\mathbf{b}) - \frac{1}{2}\mathbf{s}_{\boldsymbol{\theta}}^\dagger\mathbf{K}^{-1}\mathbf{s}_{\boldsymbol{\theta}}\right]\right\} \tag{44}$$

$$= \exp\left[\max_{\boldsymbol{\theta}}\left\{\mathbf{s}_{\boldsymbol{\theta}}^\dagger K^{-1}(\mathbf{g}-\mathbf{b}) - \frac{1}{2}\mathbf{s}_{\boldsymbol{\theta}}^\dagger\mathbf{K}^{-1}\mathbf{s}_{\boldsymbol{\theta}}\right\}\right] \tag{45}$$

The corresponding estimator is given by

$$\hat{\boldsymbol{\theta}}_I(\mathbf{g}) = \arg\max_{\boldsymbol{\theta}} \left\{ \mathbf{s}_{\boldsymbol{\theta}}^{\dagger}\mathbf{K}^{-1}(\mathbf{g}-\mathbf{b}) - \frac{1}{2}\mathbf{s}_{\boldsymbol{\theta}}^{\dagger}\mathbf{K}^{-1}\mathbf{s}_{\boldsymbol{\theta}} \right\} \tag{46}$$

Note that this is not a linear estimator due to the maximization step.

These last equations bear a certain similarity to a scanning Hotelling observer. There are differences however. The standard scanning Hotelling observer drops data independent terms and uses the test statistic

$$T_H(\mathbf{g}) = \max_{\boldsymbol{\theta}} \left\{ \mathbf{s}_{\boldsymbol{\theta}}^{\dagger}\mathbf{K}^{-1}\mathbf{g} \right\} \tag{47}$$

and estimator

$$\hat{\boldsymbol{\theta}}_H(\mathbf{g}) = \arg\max_{\boldsymbol{\theta}} \left\{ \mathbf{s}_{\boldsymbol{\theta}}^{\dagger}\mathbf{K}^{-1}\mathbf{g} \right\} \tag{48}$$

This procedure is not optimal in the EROC sense unless the dropped terms are independent of the parameters that we are trying to estimate. This could occur, for example, if the background is uniform and the noise is stationary.

4. CONCLUSIONS

The EROC curve is obviously closely related to the LROC curve. The new ideas presented here are the generalization to arbitrary parameters, the introduction of an arbitrary utility function and the reinterpretation of the ordinate as the expected utility for the true-positive cases. The relation between AEROC and 2AFC studies is new. The formulas for the ideal EROC observer are easy generalizations of those for the ideal LROC observer. The connection between ideal AEROC when the utility function is positive and ideal AUC, and the bounds and approximations derived from this connection, are new.

The AEROC is a figure of merit that can be used to measure the performance of an observer for any task which combines detection and estimation. We have shown how to estimate this figure of merit from 2AFC tests. The ideal EROC observer has a simple analytical form and can be related to many common methods of estimation. Finally, the ideal AEROC can be calculated with methods that have already been developed for the ideal AUC. Just as with the ideal AUC, the approximations and bounds given above, as well as others to be detailed elsewhere, can be used to check the bias in these calculations.

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